MATH 20D: Differential Equations Spring 2023
Homework 2

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Make sure you show all your workings.
Points will be awarded for clear explanations, not just for arriving at the correct solution.
Remember to list the sources you used when completing the assignment.
Below *NSS* is used to reference the text *Fundamentals of Differential Equations (9th edition) by Nagle, Saff, Snider*

Question (1).
(a) Solve the initial value problem

$$\frac{dy}{dx} = y^{1/3}, \qquad y(0) = 0.$$

on the domain $[0, \infty)$. *Be sure to find all solutions.*
(b) Solve the initial value problem

$$\frac{dy}{dx} = y^2 - 3y + 2 \tag{0.1}$$

on the domain $(-\infty, \infty)$ *subject to the initial conditions (i)* $y(0) = 1$ *(ii)* $y(0) = 3/2$. *Does the ODE (0.1) have a solution on* $(-\infty, \infty)$ *satisfying* $y(0) = 0$*? Justify.*
(c) Solve the initial value problem

$$\frac{dy}{dx} = \frac{1}{2}(1 + y^2)\cos(x), \qquad y(0) = 0.$$

on the domain $(-\infty, \infty)$. *You must explain why your solution is defined on* $(-\infty, \infty)$.

Question (2). *Use the method of integrating factors to solve the differential equations below subject to the stated initial conditions. Be sure to give the domains of your solutions, you should choose these domains so that they are as large as possible.*
(a) Solve $x\frac{dy}{dx} + y = x\cos(x^2)$ *subject to the initial condition*

$$y(\sqrt{\pi}) = 1.$$

(b) Solve $x\frac{dy}{dx} = y + x^2\sin(x)$ *subject to the initial condition*

$$y(\pi) = 0.$$

(c) *Solve* $x^2 \frac{dy}{dx} + 3xy = x^4 \log(x) + 1$ *subject to the initial condition*

$$y(1) = 1.$$

Question (3. *NSS 2.3.25*). *Show that the solution to the initial value problem*

$$\frac{dy}{dx} + 2xy = 1, \qquad y(2) = 1$$

on the interval $(-\infty, \infty)$ *can be expressed as*

$$y(x) = e^{-x^2}\left(e^4 + \int_2^x e^{t^2}\,dt\right).$$

Question (4. *NSS 2.3.31 and 2.3.32*). *(a) Solve the initial value problem*

$$\frac{dy}{dx} + P(x)y = x, \qquad y(0) = 1$$

where

$$P(x) = \begin{cases} 1, & 0 \le x \le 2, \\ 3, & x > 2 \end{cases}.$$

Sketch the graph of your solution from $x = 0$ *to* $x = 5$. Hint: Construct your solution so that it is continuous at $x = 2$

(b) *Solve the initial value problem*

$$\frac{dy}{dx} + 2y = Q(x), \qquad y(0) = 0$$

where

$$Q(x) = \begin{cases} 2, & 0 \le x \le 3 \\ -2, & x > 3. \end{cases}$$

Sketch the graph of the solutions from $x = 0$ *to* $x = 7$. Hint: Construct your solution so that it is continuous at $x = 3$

Question (5. *NSS 2.3.35*). *Suppose a brine containing 0.2kg of salt per liter runs into a tank initially filled with 500 L of water containing 5 kg of salt. The brine enter the tank at a rate of 5 L/min. The mixture, kept uniform by stirring, is flowing out at the rate of 5 L/min.*

(a) *Find the concentration in kilograms per liter, of salt in the tank after 10 min.*

(b) *After 10 min a leak develops in the tank and an additional liter per minute of mixture flows out of the tank. What will the concentration, in kilograms per liter, of salt in the tank 20 min after the leak develops?*

Question (6. *NSS 2.3.39*). *The temperature* T *(in units of* $100°F$*) of a university classroom on a cold winter day varies with t (in hours) as*

$$\frac{dT}{dt} = \begin{cases} 1 - T, & \text{if the hearing unit is ON} \\ -T, & \text{if the heating unit is OFF.} \end{cases}$$

Suppose $T = 0$ *at 9:00 A.M., the unit is ON from 9-10 A.M., OFF from 10-11 A.M., ON from 11 A.M.-noon, and so on for the rest of the day. How warm will the classroom be at noon? Give you answers to the nearest degree fareinheit.*